

A Comparative Study Of The Effect Of Control Strategies In The Transmission Of Malaria And Dengue

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Abstract

In this paper we carry out a comparative study of an SEIRS model for vector borne diseases, taking awareness about the disease as a factor. The comparison is done using MATLAB. The susceptible human population is divided into aware and unaware susceptible population. Over a long period of time, aware susceptible population is more in dengue than in malaria. Also, the infected population is more in dengue than in malaria. Among the various control strategies used to control the population of the mosquitoes, the most efficient control measure is the treatment of infected humans by drugs. This treatment of drug is more effective in dengue than in malaria.

Keywords: SEIRS Model; Reproduction Number; Local Stability; Optimal Control Theory; Numerical Simulation.

1. Introduction

The spread of the vector - borne diseases such as malaria and dengue is entirely governed by the interaction between the humans and mosquitoes. The infection spreads when there is an increase in the mosquito population. Hence the mosquito population has to be controlled.

In order to control the mosquito population, insecticides and indoor residual spraying are used. However, they toxify the breeding sites of the mosquitoes and the aquatic habitats of the larva. Moreover, the use of these also have a negative effect on the health of the humans. Hence harmless controls such as use of mosquito nets should be used to prevent the humans from coming in contact with the mosquitoes. However, these control measures are not being used in many of the rural and semi-rural areas and hence these diseases are much prevalent in these places.

One of the reasons for these control measures not being used is the ignorance of the people about the spread of these diseases. Hence it is absolutely essential to make the people aware about the reasons and causes of these diseases. Lots of efforts are being taken to create awareness about these diseases. Global efforts have been taken to make people aware of malaria and dengue. The effect of awareness in controlling the spread of the diseases has been modelled mathematically and studied for malaria and dengue [1, 6, 16,17] and the effect of the controls on them have been observed for each of these disease separately [2, 3, 4,5, 7, 8, 9, 11,12,14]. However, a comparative study of the transmission dynamics of both malaria and dengue has not been carried out. In this paper, we compare the transmission dynamics of malaria and dengue by making use of the model developed and analysed in [13].

The paper is organised as follows: In section 2 we first recall the SEIRS model. The existence of the equilibrium points and their stability is discussed in section 3. A comparative study of the effect of the various control strategies in the transmission of malaria and dengue is carried out in section 4 and section 5 is the conclusion.

2 The SEIRS Model

The following system of non-linear differential equations define the SEIRS model with control strategies [13].

$$\begin{aligned}
 \frac{dS_{H1}}{dt} &= \Lambda_1 - \mu_H S_{H1} - (1 - u_1)\beta_1 S_{H1} I_M - (1 - k)\gamma I_H \\
 \frac{dS_{H2}}{dt} &= \delta S_{H1} - \mu_H S_{H2} - (1 - u_1)\beta_2 S_{H2} I_M + k\gamma I_H \\
 \frac{dE_H}{dt} &= (1 - u_1)\beta_1 S_{H1} I_M + (1 - u_1)\beta_2 S_{H2} I_M - \mu_H E_H - \eta_H E_H \\
 \frac{dI_H}{dt} &= \eta_H E_H - \gamma I_H - \alpha_1 I_H - \mu_H I_H \\
 \frac{dS_M}{dt} &= \Lambda_2 - (1 - u_1)\beta_3 S_M I_H - \alpha_2 S_M - \mu_M S_M - (1 - w)\mu_3 S_M \\
 \frac{dI_M}{dt} &= (1 - u_1)\beta_3 S_M I_H - \alpha_2 I_M - \mu_M I_M - (1 - w)\mu_3 S_M
 \end{aligned} \tag{1}$$

where S_{H1} = aware susceptible human population

S_{H2} = unaware susceptible human population

E_H = exposed human population

I_H = infected human population

β_1 = contact rate of unaware susceptible humans with infective mosquitoes

Λ_1 = birth rate of human population

δ = rate of transfer of unaware susceptible individual to aware susceptible class

β_2 = contact rate of aware susceptible humans with infective mosquitoes

η_H = rate of progression of humans from the exposed to the infectious class

μ_H = natural death rate of the human population

α_1 = disease related death rate of the human population

γ = rate of progression of humans from the infected class to the susceptible class after recovery

k = a fraction of recovered persons going to the aware class

S_M = susceptible mosquito population

I_M = infected mosquito population

Λ_2 = recruitment rate of mosquitoes

β_3 = contact rate of infected human with susceptible mosquitoes

α_2 = death rate of mosquitoes due to control measures

μ_M = natural death rate of mosquito population

The three control strategies are as follows:

$u_1(t)$ is the use of bed nets for personal protection,

$u_2(t)$ is the treatment with drug of infected individuals and

$u_3(t)$ is the insecticides sprayed on the breeding grounds of mosquitoes.

Further if a_2 is the drug efficacy use for treatment and a_3 is insecticide efficacy at reducing mosquito population, we have $0 \leq u_1 \leq 1$, $0 \leq u_2 \leq a_2$ and $0 \leq u_3 \leq a_3$.

Moreover $(1 - w)$ is the fraction of mosquito population reduced, and hence the mosquitoes are reduced at the rate $(1 - w)u_3$.

The feasible solution set for model (1) given by

$$\Omega = \{(S_{H1}, S_{H2}, E_H, I_H, S_M, I_M) \in R^6: (S_{H1}, S_{H2}, E_H, I_H, S_M, I_M) \geq 0; 0 \leq N_H \leq \frac{\Lambda_1}{\mu_H + \delta}; 0 \leq N_M \leq \frac{\Lambda_2}{\mu_M + \alpha_2}\}$$

is positively invariant and mathematically well posed in the domain Ω .

3 Existence of the equilibrium points and stability

The two equilibrium points, namely disease-free equilibrium point and the endemic equilibrium

point exist and they are locally stable.

Theorem 3.1 The SEIRS model has two equilibrium points which are given as follows:

1. The disease-free equilibrium point is given by

$$E_1 = (S'_{H1}, S'_{H2}, 0, 0, S'_M, 0) = \left(\frac{\Lambda_1}{\mu_H + \delta}, \frac{\delta \Lambda_1}{(\mu_H + \delta) \mu_H}, 0, 0, \frac{\Lambda_2}{\mu_M + \alpha_2}, 0 \right).$$

2. The endemic equilibrium point by $E_2 = (S^*_{H1}, S^*_{H2}, E^*_H, I^*_H, S^*_M, I^*_M)$ exists when

$$R_0^* = \frac{\Lambda_2 \eta \beta_3 (\beta_1 S^*_{H1} + \beta_2 S^*_{H2})}{(\mu_H + \eta)^2 (\alpha_1 + \gamma + \mu_H) (\mu_M + \eta)} > 1$$

Theorem 3.2 Let $R_0 = \frac{\eta \beta_3 S'_M (\beta_1 S'_{H1} + \beta_2 S'_{H2})}{(\mu_H + \eta) (\alpha_1 + \gamma + \mu_H) (\mu_M + \alpha_2)}$. Then,

1. The disease-free equilibrium point $E_1 = (S'_{H1}, S'_{H2}, 0, 0, S'_M, 0) = \left(\frac{\Lambda_1}{\mu_H + \delta}, \frac{\delta \Lambda_1}{(\mu_H + \delta) \mu_H}, 0, 0, \frac{\Lambda_2}{\mu_M + \alpha_2}, 0 \right)$ is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

2. The endemic equilibrium point is given by $E_2 = (S^*_{H1}, S^*_{H2}, E^*_H, I^*_H, S^*_M, I^*_M)$ exist and is stable when $R_0 > 1$.

The existence and local stability given in Theorem 3.1 and Theorem 3.2 can be proved on similar lines of [17].

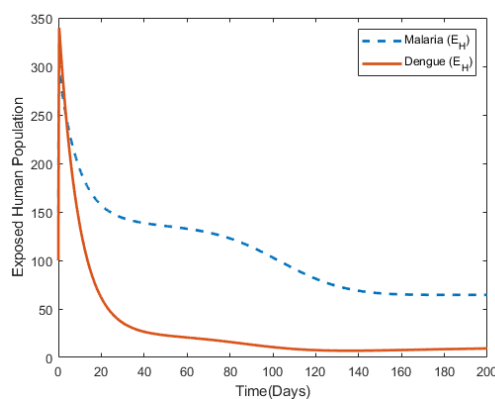
4 Comparative study of malaria and dengue by simulation

In this section we compare the exposed and infected population and then discuss the transmission dynamics of malaria and dengue for different controls. The parameters used in the simulation are given in Table. The numerical simulations are conducted using MATLAB.

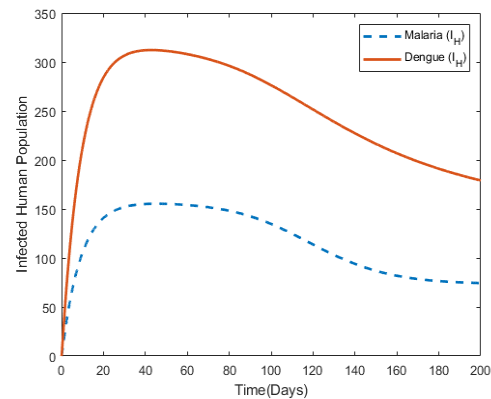
Table: Numerical values of the parameters for malaria and dengue

Parameters	Symbols	Malaria Values (days ⁻¹)	Dengue Values (days ⁻¹)
Contact rate of unaware susceptible humans with infectious mosquitoes	β_1	0.03	0.01
Contact rate of unaware susceptible humans with infectious mosquitoes	β_2	0.03	0.01
Rate of progression of humans from the exposed to the infectious state	η_H	0.058	0.1
Rate of progression of humans from the infected to the recovered state	γ	0.05	0.007
Disease induced death rate of humans	α_1	0.05	0.001
Contact rate of infected human with susceptible mosquitoes	β_3	0.09	0.75
Natural death rate of mosquitoes	μ_M	0.0667	0.071
A fraction of mosquito population reduced	w	0.85	0.85
Disease induced death rate of mosquitoes	α_2	0.05	0.01

We further assume that $\Lambda_1 = 0.00011$, $\mu_H = 0.0000457$, $k = 0.6$, $\delta = 0.002$, $\Lambda_2 = 0.071$ and $p = 0.85$ for both malaria and dengue. The values of the other parameters for malaria are taken from [3] and [10] and for dengue from [6], [9], [16], [17] and [18].

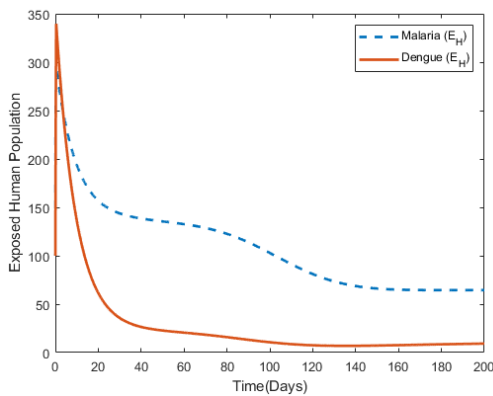


(a)

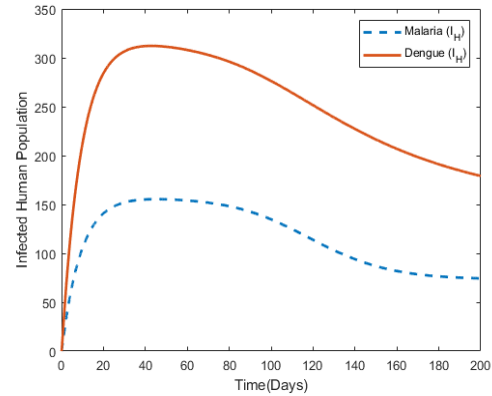


(b)

Figure 1: Comparison of the transmission of malaria and dengue in (a) unaware susceptible population (b) aware susceptible population.

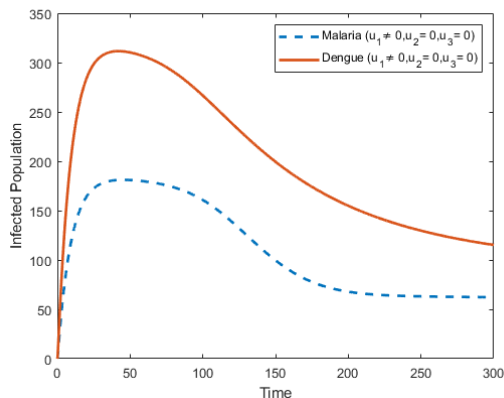


(a)

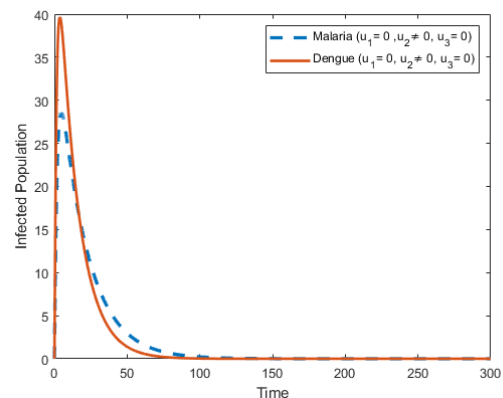


(b)

Figure 2: Comparison of the transmission of malaria and dengue in (a) exposed population (b) infected population.



(a)



(b)

Figure 3: Comparison between infected population of malaria and dengue when (a) $u_1 \neq 0, u_2 = u_3 = 0$ (b) $u_1 = 0, u_2 \neq 0, u_3 = 0$.

In Figures 1 and 2, we compare the susceptible, exposed and infected population of malaria and dengue. We compare the unaware susceptible population and the aware susceptible population in Figures 1(a) and 1(b) respectively. It can be seen from the figures that the unaware susceptible

population is more in malaria than in dengue when no control measures are used ($u_1 = u_2 = u_3 = 0$). However, in the case of aware susceptible population, after a certain period of time, there are more people aware of dengue than malaria. There are more persons exposed to malaria than to dengue (Figure 2(a)). Hence the persons infected with dengue are more than the persons infected with malaria. It is also illustrated in Figure 2(b). This is because the incubation period in malaria is more than that in dengue and due to this dengue spreads faster than malaria.

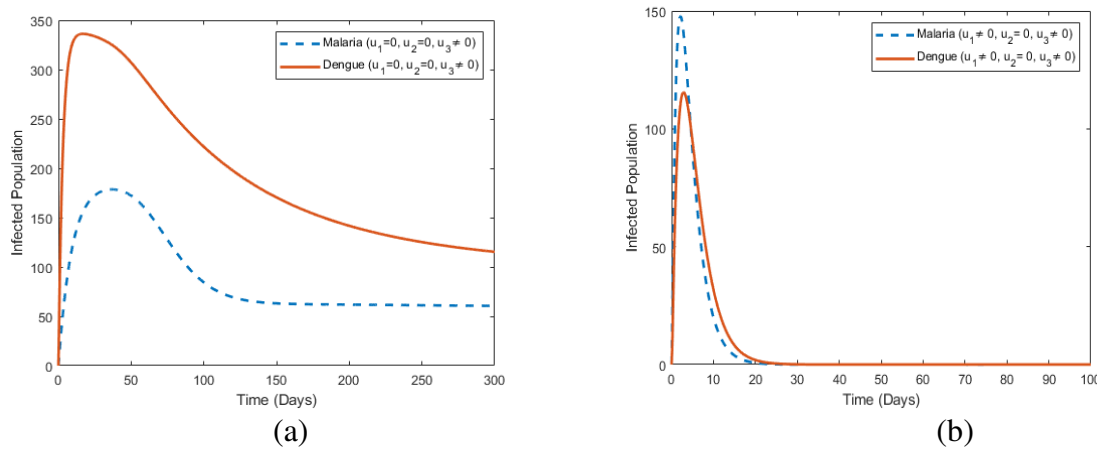


Figure 4: Comparison between infected population of malaria and dengue when (a) $u_1 = u_2 = 0, u_3 \neq 0$ (b) $u_1 \neq 0, u_2 = 0, u_3 \neq 0$.

In Figures 3 and 4, we compare the infected population of humans when various control measures are taken. It can be seen from Figure 2(b) that the infected population is more in dengue than in malaria and hence even when control measures are taken, the same holds. In Figure 5, we see the effect of treatment on both malaria and dengue. It can be observed that drug treatment has a greater effect on dengue than in malaria.

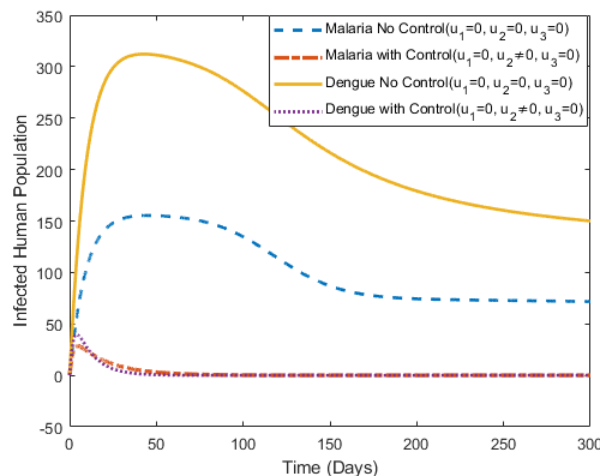


Figure 5: Comparison between infected population of malaria and dengue with u_2 (treatment with drug) control and no control.

5 Conclusion

In this article, we recalled an SEIRS model for the human population in which the susceptible population is divided into two compartments as unaware susceptible population and aware susceptible population .and use this model to compare the transmission dynamics of malaria and dengue. Over a long period of time, aware susceptible population is more in dengue than in malaria. The infected population is more in dengue than in malaria. This is because dengue spreads faster than malaria and tends to become epidemic whereas malaria is endemic. We have used various control measures such as bed nets, insecticides and treatment by drugs to see their effect on the infected population. The most efficient control measure is the treatment of infected humans by drugs. The treatment of drug is more effective in dengue than in malaria. Control measures to reduce the interaction of humans and mosquitoes also bring down the infected human population considerably in both malaria and dengue. It is very much essential that the people be made aware of the diseases even before they become infected by the disease rather than they become aware of the disease after being infected by it. Awareness helps them to take precautions and use the various control measures to prevent them from mosquito bites.

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