A New Optimal Complete Matching of Edges with Minimum Cost by Ranking Method for Solving ω-Type -2 Fuzzy Linear Sum Assignment Problem

A. Nagoor Gani[1]
PG & Research Department of Mathematics Jamal Mohamed College (Autonomous)
(Affiliated to Bharathidasan University) Tiruchirappalli – 620 020, Tamilnadu, India. E-Mail: ganijmc@yahoo.co.in[1]

T. Shiek Pareeth[2]
PG & Research Department of Mathematics Jamal Mohamed College (Autonomous)
(Affiliated to Bharathidasan University) Tiruchirappalli – 620 020, Tamilnadu,
India. E-Mail: shiekpareeth.t@gmail.com[2]

Abstract— In this paper, we discuss minimum and maximum membership values of ω- type-1 trapezoidal fuzzy number ($\omega - T1TrFN$), lower and upper membership functions of ω- type- 2 trapezoidal fuzzy number ($\omega - T2TrFN$) and find a new optimal complete matching of edges with minimum reduced cost by using ranking method for solving ω-Type-2 trapezoidal fuzzy linear sum assignment problem ($\omega - T2TrFLSAP$). Sometimes, we obtain multiple matched columns or single matched column and get corresponding optimal solution. If we get multiple matched columns and get corresponding partial optimal solution, then we proceed to each step in bipartite edges through interchange unassigned matching to new matching edges and corresponding dual variable's solution is updated from a shortest alternating path to get new optimal complete matching edges with minimum reduced cost.

Keywords— ω- trapezoidal fuzzy number; ω-type-1 trapezoidal fuzzy number ($\omega - T1TrFN$) ; ω –type-2 trapezoidal fuzzy number ($\omega - T2TrFN$); bipartite graph; alternating path; ranking method.

I INTRODUCTION

Zadeh introduced Fuzzy Set concept in 1965 [12]. Cheng, proposed a new approach for ranking fuzzy numbers by distance method in 1998. Mendel, J.M, John and Liu, discussed interval type-2 fuzzy logical systems made simple in 2006 [5].Mitchell discussed Ranking type-2 fuzzy numbers [6]. Chen and Sanguansat proposed analyzing fuzzy risk based on a new fuzzy ranking method between generalized fuzzy numbers in 2011 [4]. Amarpreet kaur and Amit kumar introduced a new method for solving fuzzy transportation problems using ranking functions in 2011 [1]. In 2013 Aminifar and Marzuki proposed uncertainty in interval type-2 fuzzy systems. Moslem Javanmard and Hassan Mishmast Nehi, solving method for fuzzy linear programming with interval type-2 fuzzy numbers in 2017 [9]. Moslem Javanmard and Hassan Mishmast Nehi discussed Rankings and operations for interval type-2 fuzzy numbers: a review and some new methods [10]. Nagoor gani. A and shiek pareeth.T compute Feasible degenerate pivoting and optimal non-degenerate pivoting for solving fuzzy linear sum assignment problems in 2018 [11]. Nagoor gani.A and shiek pareeth.T proposed a new modified optimal perfect matching in partial feasible matching for solving fuzzy linear sum assignment problems in 2018 [12]; Nagoor gani.A and shiek pareeth.T proposed a spread out of new partial feasible and optimal perfect matching for solving interval α-cut fuzzy linear sum bottleneck assignment problem in 2020 [13].

we discuss, assign each machine to a job with minimum reduced cost in that job by using ranking method of lower and upper membership function of the ω-Type-2 trapezoidal fuzzy linear sum assignment problem ($\omega - T2TrFLSAP$) and obtain new optimal complete matching of edges.

II PRELIMINARIES

We present here membership function of ω- trapezoidal fuzzy number (ω-TrFN) and minimum and maximum membership value of ω- type-1 trapezoidal fuzzy number ($\omega - T1TrFN$),lower and upper membership function of ω- type-2 trapezoidal fuzzy number ($\omega - T2TrFN$).

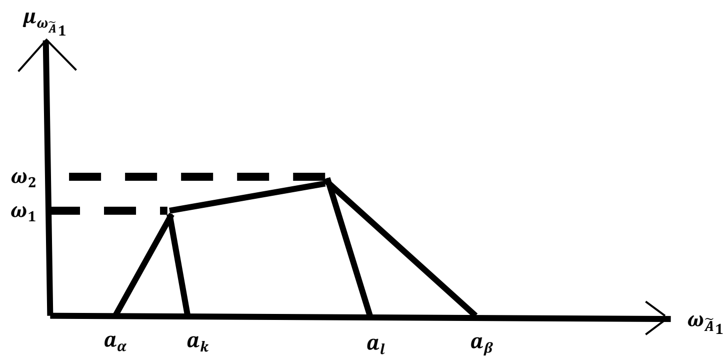
Definition 2.1: A fuzzy number $\omega_{\tilde{A}}$ is said to be ω- trapezoidal fuzzy number, then the following membership function is

$$\mu_{\omega_{\tilde{A}}} = \begin{cases} \omega\left(\frac{x-a_\alpha}{a_k-a_\alpha}\right) & if \ \ a_\alpha \le x \le a_k \\ \omega & if \ \ a_k \le x \le a_l \\ \omega\left(\frac{a_\beta - x}{a_\beta - a_l}\right) & if \ \ a_l \le x \le a_\beta \end{cases}$$

Where $\omega \in (0,1)$

Definition 2.2: A ω-type 1 trapezoidal fuzzy number($\omega - T1TrFN$) is denoted as $\omega_{\tilde{A}_1} = (a_\alpha, a_k, a_l, a_\beta, \omega_1, \omega_2)$ and ω_1 is minimum membership value of type1 trapezoidal fuzzy number($T1TrFN$) and ω_2 is maximum membership value of type1 trapezoidal fuzzy number($T1TrFN$) if the following membership function

$$\mu_{\omega_{\tilde{A}_1}} = \begin{cases} \omega_1\left(\frac{x-a_\alpha}{a_k-a_\alpha}\right) & if \ a_\alpha \le x \le a_k \\ \omega_1 + \left(\frac{x-a_k}{a_l-a_k}\right)(\omega_2-\omega_1) & if \ a_k \le x \le a_l \\ \omega_2\left(\frac{a_\beta - x}{a_\beta - a_l}\right) & if \ a_l \le x \le a_\beta \end{cases} \quad \text{Where} \quad \omega_1 < \omega_2$$

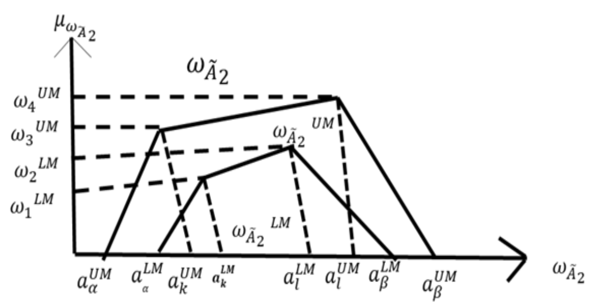


Definition 2.3: A fuzzy number $\omega_{\tilde{A}_2}$ is said to be ω-type 2 trapezoidal fuzzy number ($\omega - T2TrFN$) and is defined as, $\omega_{\tilde{A}_2}{}^{LM} = (a_\alpha^{LM},\ a_k^{LM},\ a_l^{LM}, a_\beta^{LM}, {\omega_1}^{LM}, {\omega_2}^{LM})$ and , $\omega_{\tilde{A}_2}{}^{UM} = (a_\alpha^{UM},\ a_k^{UM},\ a_l^{UM}, a_\beta^{UM}, {\omega_3}^{UM}, {\omega_4}^{UM})$ are lower and upper membership function of ω-type 2 trapezoidal fuzzy number,(${\omega_1}^{LM}$, ${\omega_2}^{LM}$) and (${\omega_3}^{UM}$, ${\omega_4}^{UM}$) are lower and upper membership value of ω-type 2 trapezoidal fuzzy number ($\omega - T2TrFN$), if the following membership function

$$\mu_{\omega_{\tilde{A}_2}{}^{LM}} = \begin{cases} {\omega_1}^{LM}\left(\frac{x-a_\alpha^{LM}}{a_k^{LM}-a_\alpha^{LM}}\right) & if \ a_\alpha^{LM} \le x \le a_k^{LM} \\ {\omega_1}^{LM} + \left(\frac{x-a_k^{LM}}{a_l^{LM}-a_k^{LM}}\right)({\omega_2}^{LM}-{\omega_1}^{LM}) & if \ a_k^{LM} \le x \le a_l^{LM} \\ {\omega_2}^{LM}\left(\frac{a_\beta^{LM}-x}{a_\beta^{LM}-a_l^{LM}}\right) & if \ a_l^{LM} \le x \le a_\beta^{LM} \end{cases}$$

and

$$\mu_{\omega_{\tilde{A}_2}{}^{UM}} = \begin{cases} {\omega_3}^{UM}\left(\frac{x-a_\alpha^{UM}}{a_k^{UM}-a_\alpha^{UM}}\right) & if \ a_\alpha^{UM} \le x \le a_k^{UM} \\ {\omega_3}^{UM} + \left(\frac{x-a_k^{UM}}{a_l^{UM}-a_k^{UM}}\right)({\omega_4}^{UM}-{\omega_3}^{UM}) & if \ a_k^{UM} \le x \le a_l^{UM} \\ {\omega_4}^{UM}\left(\frac{a_\beta^{UM}-x}{a_\beta^{UM}-a_l^{UM}}\right) & if \ a_l^{UM} \le x \le a_\beta^{UM} \end{cases}$$



Where,
$a_\alpha^{UM} < a_\alpha^{LM} < a_k^{UM} < a_k^{LM} < a_l^{LM} < a_l^{UM} < a_\beta^{LM} < a_\beta^{UM}$ and ${\omega_1}^{LM} < {\omega_2}^{LM} < {\omega_3}^{UM} < {\omega_4}^{UM}$.

III. ARITHMETIC OPERATIONS OF ω-TYPE-2 TRAPEZOIDAL FUZZY NUMBER ($\omega - T2TrFN$)

Let , $\omega_{\tilde{A}_2}{}^{LM} = (a_\alpha^{LM},\ a_k^{LM},\ a_l^{LM}, a_\beta^{LM}, \omega_1{}^{LM}, \omega_2{}^{LM})$, $\omega_{\tilde{A}_2}{}^{UM} = (a_\alpha^{UM},\ a_k^{UM},\ a_l^{UM}, a_\beta^{UM}, \omega_3{}^{UM}, \omega_4{}^{UM})$; $\omega_{\tilde{B}_2}{}^{LM} = (b_\alpha^{LM},\ b_k^{LM},\ b_l^{LM}, b_\beta^{LM}, \omega_1{}^{LM}, \omega_2{}^{LM})$ $\omega_{\tilde{B}_2}{}^{UM} = (b_\alpha^{UM},\ b_k^{UM},\ b_l^{UM}, b_\beta^{UM}, \omega_3{}^{UM}, \omega_4{}^{UM})$ are two ω –type-2 trapezoidal fuzzy numbers ,then the following operations.

i) **Addition:**

$\omega_{\tilde{A}_2} \oplus \omega_{\tilde{B}_2} = (\omega_{\tilde{A}_2}{}^{LM} \oplus \omega_{\tilde{B}_2}{}^{LM},\ \omega_{\tilde{A}_2}{}^{UM} \oplus \omega_{\tilde{B}_2}{}^{UM}) = ((a_\alpha^{LM} \oplus b_\alpha^{LM},\ a_k^{LM} \oplus b_k^{LM},\ a_l^{LM} \oplus b_l^{LM},\ a_\beta^{LM} \oplus b_\beta^{LM}, \min\{\omega_1{}^{LM}, \omega_2{}^{LM})\ \min(\omega_3{}^{LM}, \omega_4{}^{LM}\}), (a_\alpha^{UM} \oplus b_\alpha^{UM},\ a_k^{UM} \oplus b_k^{UM},\ a_l^{UM} \oplus b_l^{UM}, a_\beta^{UM} \oplus b_\beta^{UM}, \min\{\omega_1{}^{UM}, \omega_2{}^{UM})\ \min(\omega_3{}^{UM}, \omega_4{}^{UM}\}$.

(**ii) Subtraction:**

$\omega_{\tilde{A}_2} \Theta\, \omega_{\tilde{B}_2} = (\omega_{\tilde{A}_2}{}^{LM}\, \Theta\, \omega_{\tilde{B}_2}{}^{LM},\ \omega_{\tilde{A}_2}{}^{UM} \Theta\ \omega_{\tilde{B}_2}{}^{UM}) = ((a_\alpha^{LM} - b_\beta^{LM},\ a_k^{LM} - b_l^{LM},\ a_l^{LM} - b_k^{LM},\ a_k^{LM} - b_\alpha^{LM}, \min\{\omega_1{}^{LM}, \omega_2{}^{LM})\ \min(\omega_3{}^{LM}, \omega_4{}^{LM})), (a_\alpha^{UM} - b_\beta^{UM},\ a_k^{UM} - b_l^{UM},\ a_l^{UM} - b_k^{UM}, a_k^{LM} - b_\alpha^{LM}, \min(\omega_1{}^{UM}, \omega_2{}^{UM})\ \min(\omega_3{}^{UM}, \omega_4{}^{UM}))$.

(iii) Scalar Multiplications:

$\lambda\, \omega_{\tilde{A}_2} = (\omega_{\tilde{A}_2}{}^{LM},\ \omega_{\tilde{A}_2}{}^{UM}) = (\lambda a_\alpha^{LM}, \lambda a_k^{LM}, \lambda a_l^{LM}, \lambda a_\beta^{LM}, \omega_1{}^{LM}, \omega_2{}^{LM})\ (\lambda a_\alpha^{UM}, \lambda a_k^{UM}, \lambda a_l^{UM}, \lambda a_\beta^{UM}, \omega_3{}^{UM}, \omega_4{}^{UM})$ .If $\lambda \geq 0$,

$\lambda\, \omega_{\tilde{A}_2} = (\omega_{\tilde{A}_2}{}^{LM},\ \omega_{\tilde{A}_2}{}^{UM} = (\lambda a_\beta^{LM}, \lambda a_l^{LM}, \lambda a_k^{LM}, \lambda a_\alpha^{LM}, \omega_1{}^{LM}, \omega_2{}^{LM})\ (\lambda a_\beta^{UM}, \lambda a_l^{UM}, \lambda a_k^{UM}, \lambda a_\alpha^{UM}, \omega_3{}^{UM}, \omega_4{}^{UM})$ If $\lambda \leq 0$

IV. A NEW RANKING METHOD OF ω-TYPE-2 TRAPEZOIDAL FUZZY NUMBER ($\omega - T2TrFN$)

The lower membership function of ω-type-2 trapezoidal fuzzy number is denoted as $\omega_{\tilde{A}_2}{}^{LM}$ and upper membership function of ω-type-2 trapezoidal fuzzy number is denoted as $\omega_{\tilde{A}_2}{}^{UM}$; where $\omega_{\tilde{A}_2}{}^{LM} = (a_\alpha^{LM},\ a_k^{LM},\ a_l^{LM}, a_\beta^{LM}, \omega_1{}^{LM}, \omega_2{}^{LM})$ and $\omega_{\tilde{A}_2}{}^{UM} = (a_\alpha^{UM},\ a_k^{UM},\ a_l^{UM}, a_\beta^{UM}, \omega_3{}^{UM}, \omega_4{}^{UM})$. The ranking method of lower membership function of the ω-type-2 trapezoidal fuzzy number($\omega_{\tilde{A}_2}{}^{LM}$) is defined as $R_{\omega_{\tilde{A}_2}{}^{LM}}$ and The ranking method of upper membership function of the ω-type-2 trapezoidal fuzzy number is defined as $R_{\omega_{\tilde{A}_2}{}^{UM}}$.

V .THE NEW RANK OF OF ω-TYPE-2 TRAPEZOIDAL FUZZY NUMBER ($\omega - T2TrFN$)

$$R_{\omega_{\tilde{A}_2}} = \frac{1}{2}\left[R_{\omega_{\tilde{A}_2}{}^{LM}} \min(\omega_1{}^{LM},\ \omega_2{}^{LM}) + R_{\omega_{\tilde{A}_2}{}^{UM}} \max(\omega_3{}^{UM},\ \omega_4{}^{UM})\right]$$

Where $R_{\omega_{\tilde{A}_2}{}^{LM}} \min(\omega_1{}^{LM},\ \omega_2{}^{LM}) = \left(\frac{(a_\alpha^{LM} + a_k^{LM} + a_l^{LM} + a_\beta^{LM}) \min(\omega_1{}^{LM}, \omega_2{}^{LM})}{4}\right)$;

$R_{\omega_{\tilde{A}_2}{}^{UM}} \max(\omega_3{}^{UM},\ \omega_4{}^{UM}) = \left(\frac{(a_\alpha^{UM} + a_k^{UM} + a_l^{UM} + a_\beta^{UM}) \max(\omega_3{}^{UM}, \omega_4{}^{UM})}{4}\right)$

$$R_{\omega_{\tilde{A}_2}} = \frac{1}{2}\left[\left(\frac{(a_\alpha^{LM} + a_k^{LM} + a_l^{LM} + a_\beta^{LM})\, min(\omega_1{}^{LM},\ \omega_2{}^{LM})}{4}\right) + \left(\frac{(a_\alpha^{UM} + a_k^{UM} + a_l^{UM} + a_\beta^{UM})\, max(\omega_3{}^{UM},\ \omega_4{}^{UM})}{4}\right)\right]$$

$$R_{\omega_{\tilde{A}_2}} = \left(\frac{(a_\alpha^{LM} + a_k^{LM} + a_l^{LM} + a_\beta^{LM}) \min(\omega_1{}^{LM},\ \omega_2{}^{LM}) + (a_\alpha^{UM} + a_k^{UM} + a_l^{UM} + a_\beta^{UM}) \max(\omega_3{}^{UM},\ \omega_4{}^{UM})}{8}\right)$$

Theorem 1: If $R_{\omega_{v_0}} = \emptyset$, then Xij =1 provides an optimal complete matching of edges.

proof

First choose , $R_{\omega_{v_{12}}} \neq \emptyset$, then we proceed with the lowest possible expense for each job j ∈ $R_{\omega_{v_{12}}}$ with $R_{\omega_{\pi_j}}$= min {$R_{\omega_{c_{ij}}}$- $R_{\omega_{u_i}}$: row i is present in the tree}. pick j ∈ $R_{\omega_{v_{12}}}$ with minimum $R_{\omega_{\pi_j}}$ - $R_{\omega_{v_j}}$ and set $R_{\omega_{\pi_j}} = R_{\omega_{v_j}}$ then add new edge [i',j'] to the tree, where i' is the row vertex compute $R_{\omega_{\pi_j}}$. For each i∈U ; Xi j' =1 then we proceed to $R_{\omega_{u_i}} = R_{\omega_{c_{ij'}}}$- $R_{\omega_{v_{j'}}}$and [i,j'] add to the tree. $R_{\omega_{v_0}}$, $R_{\omega_{v_1}}$and $R_{\omega_{v_2}}$ should all be modified and hence Continue the loop until $R_{\omega_{v_0}} = \emptyset$ and then the turn paths to obtain a new optimal complete matching of edges.

VI. A NEW OPTIMAL COMPLETE MATCHING SOLUTION WITH MINIMUM REDUCED COST BY USING RANKING METHOD OF ω-TYPE- 2 FUZZY LINEAR SUM ASSIGNMENT PROBLEM ($\omega - T2FLSAP$):

We discuss a new optimal complete matching solution for solving ω-type-2 fuzzy linear sum assignment problem by using ranking method and compute each machine to a job with minimum reduced cost in that job. We introduce the following new procedure.

Step 1: If the total number of machines (M) is equal to the total number of jobs (J), then ω-type-2 fuzzy linear sum assignment problem is balanced. otherwise ω-type-2 fuzzy
linear sum assignment problem is unbalanced. if ω-type-2 fuzzy linear sum assignment problem is unbalanced, then we introduce dummy row or dummy column.
Step 2: Balanced ω-type- 2 fuzzy linear sum assignment problem is converted to ranking of balanced ω-type-2 fuzzy linear sum assignment problem.
Step 3: Let us assume $R_{\omega_{v_0}}$, $R_{\omega_{v_1}}$and $R_{\omega_{v_2}}$ are column vertices with no machine Matched, one machine matched and more than one or two machine matched. For job j =1,2,3… n do $R_{\omega_{v_j}}$ = 0.
Step 4: construction of the tree: Choose a job r ∈ $R_{\omega_{v_2}}$as the root, and set v_r =0; for each i∈U, Xir =1 then we proceed $R_{\omega_{u_i}}$= $R_{\omega_{c_{ir}}}$ the tree with the edge [i,r] such that Xir =1.
Step 5 : Select one row assigned column $R_{\omega_{v_{12}}} \neq \emptyset$ then we proceed for each job j ∈ $R_{\omega_{v_{12}}}$ with minimum reduced cost $R_{\omega_{\pi_j}}$= min {$R_{\omega_{c_{ij}}}$- $R_{\omega_{u_i}}$: row i is in the tree}.
Step 6: Select j ∈ $R_{\omega_{v_{12}}}$ with minimum $R_{\omega_{\pi_j}}$ - $R_{\omega_{v_j}}$ and set $R_{\omega_{\pi_j}}$ = $R_{\omega_{v_j}}$ then add new edge [i',j'] to the tree, where i' is the row vertex obtaining $R_{\omega_{\pi_j}}$. For each i∈U ; Xi j' =1 then
we proceed to $R_{\omega_{u_i}}$= $R_{\omega_{c_{ij'}}}$- $R_{\omega_{v_{j'}}}$and [i,j'] add to the tree.
Step 7: Select the unassigned job $R_{\omega_{j^*}} \in \overline{R_{\omega_{v_0}}}$ and set $R_{\omega_{u_{i^*}}}$ = $arg\ min_{i \in U}$ {$R_{\omega_{c_{ij^*}}}$-$R_{\omega_{u_i}}$},$R_{\omega_{v_{j^*}}}$= $R_{\omega_{c_{i^*j^*}}}$-$R_{\omega_{u_{i^*}}}$. Let P be the bipartite path $R_{\omega_{u_{i^*}}}$ connecting to a column of $R_{\omega_{v_2}}$.
Interchange unmatched edges to matched edges along path, $X_{i^*j^*}$=1.
Step 8: Update $R_{\omega_{v_0}}$, $R_{\omega_{v_1}}$and $R_{\omega_{v_2}}$. Continue the process until $R_{\omega_{v_0}} = \emptyset$ and then alternate paths to obtain a new optimal complete matching of edges.
Step 9: Stop.

A. Example:

The number of machines equal to the number of jobs, so therefore the given fuzzy linear sum assignment problem is balanced otherwise unbalanced fuzzy linear sum assignment problem The balanced ω-type-2 trapezoidal fuzzy number of fuzzy linear sum assignment problem as follows.the new ranking method of ω-type-2 trapezoidal fuzzy number($\omega - T2TrFN$) is as follows:

$\omega_{\tilde{A}_2}$ = (5,11,14,20,0.2,0.4);(2,8,17,23,0.4,0.8) → $R_{\omega_{\tilde{A}_2}}$ = 6.25 ,
$\omega_{\tilde{A}_2}$ =(8,16,20,28,0.1,0.3); (4,12,24,32,0.45,0.6) → $R_{\omega_{\tilde{A}_2}}$ = 6.30.
$\omega_{\tilde{A}_2}$ = (14,26,32,44,0.25,0.3); (8,20,38,50,0.4,0.5) → $R_{\omega_{\tilde{A}_2}}$ = 10.88
$\omega_{\tilde{A}_2}$ = (20,30,35,45,0.25,0.4); (15,25,40,50,0.6,0.8) → $R_{\omega_{\tilde{A}_2}}$ = 17.06
$\omega_{\tilde{A}_2}$ = (20,40,50,70,0.2,0.35); (10,30,60,80,0.5,0.8) → $R_{\omega_{\tilde{A}_2}}$ = 22.50
$\omega_{\tilde{A}_2}$ = (40,60,70,90,0.3,0.5); (30,50,80,100, .65, .95) → $R_{\omega_{\tilde{A}_2}}$ = 40.63
$\omega_{\tilde{A}_2}$ =(35,45,50,60,0.65,0.75) ; (30,40,55,65,0.8,0.9) → $R_{\omega_{\tilde{A}_2}}$ = 36.81
$\omega_{\tilde{A}_2}$ = (60,80,90,110,0.25,0.45) ; (50,70,100,120,0.7,0.95) → $R_{\omega_{\tilde{A}_2}}$ = 51

Machine(M)/ JOB(J)	J_1	J_2	J_3	J_4
M_1	**36.81**	51	40.63	51
M_2	**6.30**	40.63	10.88	36.81
M_3	**6.25**	22.50	22.50	51
M_4	17.06	22.50	**6.30**	6.30

Table:2

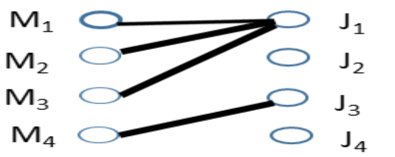


FIGURE - 1

Machine(M)/ JOB(J)	J_1	J_2	J_3	J_4
M_1	36.81	**51**	40.63	51
M_2	**6.30**	40.63	10.88	36.81
M_3	**6.25**	22.50	22.50	51
M_4	17.06	22.50	**6.30**	6.30

Table:3

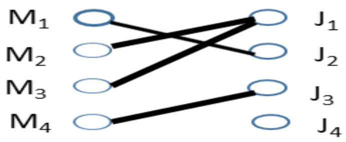


FIGURE - 2

Updated dual solution and $R_{\omega_{v_{12}}}$ ={2,3} and obtain $R_{\omega_{\pi_2}}$= min {$R_{\omega_{c_{ij}}}$- $R_{\omega_{u_i}}$= 16.25 (row 3) add new edge[3,2], update $R_{\omega_{u_1}}$ = 34.75 and $R_{\omega_{\pi_3}}$= min {$R_{\omega_{c_{ij}}}$- $R_{\omega_{u_i}}$= = 4.58 (row 2), add new matching edge [2,3]; update $R_{\omega_{u_4}}$= 1.72., add new matching edge [1,2]; Update the dual solutions is $R_{\omega_{u_i}}$ = (34.75, 6.30, 6.25,1.72); $R_{\omega_{v_j}}$ = (0, 16.25, 4.58, 4.58).select unassigned column $R_{\omega_{j^*}}$= {4} $R_{\omega_{v_{j^*}}}$=($R_{\omega_{c_{i^*j^*}}}$-$R_{\omega_{u_{i^*}}}$) = 4.58 (row 4).hence, path P= {[4,3],[2,3],[2,1]}, unassigned to assigned edges are X43=0, X23 = 1, X21= 0 , X44= 1.

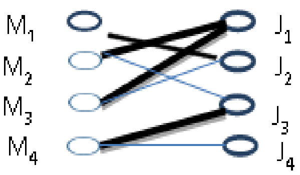


FIGURE - 3

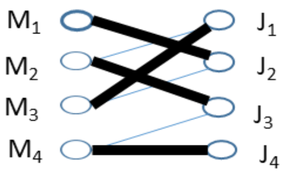


FIGURE - 4

ω-Type 2 Fuzzy Linear Sum assignment schedule is

$M_1 \to J_2$; $M_2 \to J_3$; $M_3 \to J_1$; $M_4 \to J_4$

A new Optimal complete matching solution of ranking

$\omega - T2FLSAP$ is ***51+10.88+6.25 + 6.30 = 74.43***

A new Optimal complete solution of $\omega - T2FLSAP$ is ***(87,133,156,202,0.1,0.3);(64,110,179,225,0.7,0.95)***

CONCLUSION

We use ranking of -type 2 Trapezoidal fuzzy number to assign each machine to a job with the lowest cost/time in that job for solving -type 2 fuzzy linear sum assignment problem (ω -T2FLSAP). Furthermore, each iteration updates a non-matched edge to a matched edge and the corresponding dual variable's solution by alternating path to a new optimal total matching solution.

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