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LINEAR STOCHASTIC FORECASTING MODEL WITH MOVING AVERAGE PROCESS FOR DISTURBANCES

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ABSTRACT

Forecasting methodology starts basically with model building. In forecasting models, linear regression models are widely used, attention is concerned on model specification when the model is chosen to be linear. There is considerable literature on the problem of estimating linear stochastic model with first order Moving Average (MA) disturbances. The estimation methods suggested in the literature are not computationally simple and they are not easily generalized for the higher order MA disturbances.

This article proposes a method to estimate the linear stochastic models with MA disturbances by using internally studentized residuals. Later these estimated models were implemented for the purpose of forecasting.

I. INTRODUCTION:

Applied regression analysis techniques, which originated in the twentieth century have recently gained in popularity for forecasting. The regression approach offers many valuable contributions to the solutions of the forecasting problem. Forecasting methodology starts basically with model building. The usual approach to forecasting involves choosing a forecasting model among several competing alternatives and using that model to generate forecasts for the series of interest.

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

There is considerable literature on the problem of estimating linear regression model with first order Moving Average (MA) disturbances. The methods of estimation suggested in the literature are not computationally simple. They are not easily generalized for the higher order MA disturbances.

In the present study, a method has been proposed to estimate the linear models with moving average disturbances. In this method, Internally studentized residuals have been used to estimate the parameters.

II. LINEAR REGRESSION MODEL WITH MA (1) PROCESS FOR DISTURBANCES:

Consider the linear regression model with first order moving average disturbances as:

$$\left.\begin{aligned} y_t &= x_t^1 \beta + \in_t , \\ \in_t &= u_t - \theta_1 u_{t-1} , \quad t = 1, 2, \ldots, T \end{aligned}\right\} \qquad (2.1)$$

Where y_t is the t^{th} observation on the dependent variable y ;

x_t is a kx1 vector of observations on k fixed regressors ;

β is a (kx1) vector of unknown parameters;

$\in_t$ is the disturbance term following a first order moving average scheme with unknown parameter θ_1 ; and

$$\left.\begin{aligned} \mathrm{E}(u_t) &= 0, \forall t \\ \mathrm{E}(u_t u_{t+s}) &= \sigma_u^2 , if\ s = 0 \\ &= 0, \text{ otherwise} \end{aligned}\right\} \qquad (2.2)$$

The model (2.1) can be written in the matrix notation as

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

$$Y_{Tx1} = X_{Txk}\ \beta_{kx1} + \in_{Tx1}$$

$$E(\in) = 0 \ \ and \ \ \mathrm{E}\left(\in \in^{1}\right) = \Omega = \sigma_u^2 v \qquad (2.3)$$

$$V = \begin{bmatrix} 1+\theta_1^2 & -\theta_1 & 0 \ \ldots\ldots\ 0 & 0 \\ -\theta_1 & 1+\theta_1^2 & -\theta_1 \ldots\ldots\ 0 & 0 \\ . & & & \\ . & & & \\ . & & & \\ . & & & \\ 0 & 0 & 0 \ldots\ldots 1+\theta_1^2 & -\theta_1 \\ 0 & 0 & 0 \ldots\ldots -\theta_1 & 1+\theta_1^2 \\ & & & \end{bmatrix} \qquad (2.4)$$

Here Ω is the symmetric positive definite variance – covariance matrix for T consecutive draws from an MA (1) process.

In other words, the autocovariances of MA (1) process are given by,

$$\left.\begin{aligned} \gamma_0 &= \left(1+\theta_1^2\right)\sigma_u^2 \\ \gamma_1 &= -\theta_1\,\sigma_u^2 \\ \gamma_2 &= \gamma_3 = ---= 0 \end{aligned}\right\} \qquad (2.5)$$

which gives the autocorrelation coefficients as

$$\left.\begin{aligned} \rho_1 &= \frac{-\theta_1}{1+\theta_1^2} \\ \rho_2 &= \rho_3 = \ldots = 0 \end{aligned}\right\} \qquad (2.6)$$

,

The MA (1) process may be inverted to give u_t as an infinite series in $\in_t$, $\in_{t-1}, \ldots.$ namely

$$u_t = \in_t + \theta_1 \in_{t-1} + \theta_1^2 \in_{t-2} + \ldots. \qquad (2.7)$$

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

that is, $\in_t = -\theta_1 \in_{t-1} - \theta_1^2 \in_{t-2} - \ldots\ldots + u_t$ (2.8)

The process (2.8) is an $AR(\infty)$ series and the partial autocorrelations do not cut off but damp toward zero; but, the autocorrelation coefficients are zero after the first autocorrelation coefficient. The properties of a pure MA process are the converse of those of a pure AR Process.

The autocovariance function (ACF) of an MA process cuts off after the order of the process and the partial ACF damps toward zero.

The process (2.8) only makes the sense $if \left|\theta_1\right| < 1$. This condition $\left|\theta_1\right| < 1$ is known as the 'Invertibility condition'. It is similar to the stationarity condition for an AR(1) process but stationarity of the MA(1) process ifself does not impose any condition on θ_1 .

Now, the OLS estimator of β is given by

$$\hat{\beta} = \left(x^1 x\right)^{-1} X^1 y \quad (2.9)$$

and $\operatorname{var}\left(\hat{\beta}\right) = \sigma_u^2 \left(x^1 x\right)^{-1} \left(x^1 \vee x\right) \left(x^1 x\right)^{-1}$ (2.10)

$\hat{\beta}$ is an unbiased estimator but not the best linear unbiased estimator (BLUE) for β.

Rewrite the model (2.1) or (2.3) as

$$y^* = x^* \beta + z^* \delta + u \quad (2.11)$$

Where $\delta = u_0$ and

$$\left.\begin{array}{ll} y_t^* = y_t + \theta_1 y_{t-1}^* , & y_0^* = 0 \\ x_t^* = x_t + \theta_1 x_{t-1}^* , & x_0^* = 0 \end{array}\right\} \quad (2.12)$$

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

$$z_t^* = \theta_1 z_{t-1}^* \ , \qquad z_0^* = -1$$

It follows from Pagan and Nicholls (1976) that the minimization of U^1U with respect to θ, β *and* δ is the same as the minimization of $\left[\in^1 \vee^{-1} \in\right]$ with respect to θ *and* β.

Since, Ω is the symmetric positive definite matrix, there exists a non-singular matrix p such that $pp^1 = v^{-1}$. Multiplying the both sides of the model (2.3) by p^1 gives.

$$p^1 y = p^1 X\beta + p^1 \in \qquad (2.13)$$

By applying to OLS to the transformed model (2.13), one may obtain the Generalized Least Squares (GLS) estimator for β as

$$\tilde{\beta} = \left(X^1 v^{-1} X\right)^{-1} X^1 v^{-1} y \qquad (2.14)$$

and $\operatorname{var}\left(\tilde{\beta}\right) = \sigma_u^2 \left(X^1 v^{-1} X\right)^{-1}$ (2.15)

where $\tilde{\beta}$ is the BLUE for β.

Here, the GLS estimator $\tilde{\beta}$ is obtained by minimizing

$\in^1 v^{-1} \in = \left(y - X\beta\right)^1 v^{-1} \left(y - X\beta\right)$ with respect to β.

III. LINEAR REGRESSION MODEL WITH MA(2) PROCESS FOR DISTURBANCES:

Consider the linear regression model with second order moving average disturbances as:

$$y_t = x_t^1 \beta + \in_t \qquad (3.1)$$

$$\in_t = u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} \qquad (3.2)$$

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

Where $\in_t$ is the disturbance term following a second order moving average scheme with unknown parameters $\theta_1 \; and \, \theta_2$; and

$$\left.\begin{aligned} &E(u_t) = 0, \;\; \forall t \\ &E(u_t \; u_{t+s}) = \sigma_u^2 \;, if \; s = 0 \\ &\qquad\qquad\;\; = \;0, \text{ otherwise} \end{aligned}\right\} \qquad (3.3)$$

The model (3.1) can be written in the matrix notation as

$$\left.\begin{aligned} &Y_{Tx1} = X_{Txk} \; \beta_{kx1} + \in_{Tx1} \\ &E(\in) = 0 \; and \;\; E\left(\in \in^1\right) \; = \Omega \end{aligned}\right\} \qquad (3.4)$$

Here, Ω represents the variance-covariance matrix of T consecutive draws from an MA (2) process.

$$\Omega = \left(\left(w_{ij}\right)\right), \qquad i \,, \; j = 1, 2, \ldots, T$$

Where $(i, \; j)^{th}$ element of Ω is given by

$$w_{ij} \;\; = \;\; \gamma_{|i-j|}$$

Here, γ_k is the kth order autocovariance function of MA (2) process.

$$\left.\begin{aligned} &\gamma_0 \; = \; \sigma_u^2 \left(1 + \theta_1^2 + \theta_2^2\right) \\ &\gamma_1 \; = \; \sigma_u^2 \left(-\theta_1 \; + \theta_1 \, \theta_2\right) \\ &\gamma_2 \; = \; -\sigma_u^2 \; \theta_2 \end{aligned}\right\} \qquad (3.5)$$

$$\gamma_k = 0, k > 2$$

The autocorrelation coefficients are given by

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

$$\left.\begin{aligned} \rho_1 &= \frac{-\theta_1 + \theta_1 \theta_2}{1+\theta_1^2+\theta_2^2} = \frac{-\theta_1\left(1-\theta_2\right)}{1+\theta_1^2+\theta_2^2} \\ \rho_2 &= \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} \\ \rho_k &= 0, \quad k>2 \end{aligned}\right\} \qquad (3.6)$$

Now, the OLS estimator of β is given by

$$\hat{\beta} = \left(X^1X\right)^{-1} X^1Y \qquad (3.7)$$

$$\text{and } \operatorname{var}\left(\hat{\beta}\right) = \left(X^1 X\right)^{-1} \left(X^1\Omega X\right) \left(X^1 X\right)^{-1} \qquad (3.8)$$

The BLUE of β is the GLS estimator $\tilde{\beta}$, which is given by

$$\tilde{\beta} = \left(X^1\Omega^{-1}X\right)^{-1} X^1\Omega^{-1}Y \qquad (3.9)$$

$$and \quad \operatorname{var}\left(\tilde{\beta}\right) = \left(X^1\Omega^{-1}X\right)^{-1} \qquad (3.10)$$

IV. LINEAR REGRESSION MODEL WITH MA(q) PROCESS FOR DISTURBANCES:

Consider the linear regression model with qth order moving average disturbances as:

$$y_t = x_t^1\beta + \in_t , \qquad (4.1)$$

$$\in_t = u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \ldots . - \theta_q u_{t-q} \qquad (4.2)$$

Where $\in_t$ is the disturbance term following a qth order moving average scheme with unknown parameters, $\theta_1, \theta_2, \ldots, \theta_q$;

$$\left.\begin{aligned} &\text{and } \mathrm{E}\left(u_t\right) = 0 , \forall t \\ &\mathrm{E}\left(u_t\ u_{t+s}\right) = \sigma_u^2, if\ s=0 \end{aligned}\right\} \qquad (4.3)$$

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

= 0, otherwise

The model (4.1) can be written in the matrix notation as

$$Y_{Tx1} = X_{Txk}\,\beta_{kx1} + \in_{Tx1} \tag{4.4}$$

$$\mathrm{E}(\in) = 0 \;\; and \;\; \mathrm{E}(\in\in^{1}) = \Omega \tag{4.5}$$

Here Ω represents the variance – covariance matrix of T consecutive draws from an MA (q) process.

We have, $\Omega = ((w_{ij}))$, $i, j = 1, 2, \ldots T$

Where $w_{ij} = \gamma_{|i-j|}$

$$\Omega = \begin{bmatrix} . & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & \gamma_q & . \\ . & . & . & . & . & \gamma_1 & . & . \\ . & . & . & . & \gamma_0 & . & . & . \\ . & . & . & \gamma_1 & . & . & . & . \\ . & . & \gamma_q & . & . & . & . & . \\ 0 & . & . & . & . & . & . & . \end{bmatrix}$$

Here, γ_k is the kth order autocovariance function of an MA (q) process.

$$\gamma_k = \sigma_u^2 \left(\lambda_k + \lambda_{k+1}\lambda_1 + \lambda_{k+2}\lambda_2 + \ldots + \lambda_q \lambda_{q-k} \right) \; for\; k = 0, 1, \ldots, q \tag{4.6}$$

= 0 , for k > q

Where $\lambda_0 \equiv 1$

Here, $\lambda_i = -\theta_i$, $\forall i = 1, 2, \ldots k$

The Autocorrelation coefficients are given by

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

$$\rho_k = \frac{\gamma_k}{\gamma_0} \ , \quad k = 1, 2, \ldots, q \tag{4.7}$$

By applying the GLS estimation, the BLUE of β is given by

$$\tilde{\beta} = \left(X^1 \Omega^{-1} X \right)^{-1} X^1 \Omega^{-1} Y \tag{4.8}$$

$$\text{var} \left(\tilde{\beta} \right) = \left(X^1 \Omega^{-1} X \right)^{-1} \tag{4.9}$$

In the case of MA (q), the assumption (6.4.5) can be rewritten as

$$\text{E} \left(\in \in^1 \right) = \sigma_{\in}^2 \left[I + \sum_{k=1}^{q} \rho_k c_k \right] \tag{4.10}$$

Where $\sigma_{\in}^2 = \text{ var} \left(\in_t \right) = \sigma^2 \left(1 + \theta_1^2 + \theta_2^2 + \ldots . + \theta_q^2 \right)$ (4.11)

$$\rho_k = \frac{-\theta_k + \theta_1 \theta_{k+1} + \theta_2 \theta_{k+2} + \ldots . + \theta_{q-k} \ \theta_q}{1 + \theta_1^2 + \theta_2^2 + \ldots . + \theta_q^2} \ , \quad k = 1, 2, \ldots q \tag{4.12}$$

$$= 0, \quad k > q$$

Here, C_k is a TxT matrix with 1 at $\left(t \, , t + k \right)^{th}$ place and zero else where., k = 1, 2, …… q.

The transformed regression equation corresponds to (6.2.1) can be written as

$$y^* = X^* \beta + z^* \delta + U \tag{4.13}$$

Where $\delta = \left[u_0 \ u_{-1} \ldots . u_{-(q-1)} \right]^1$ is a (qx1) vector

$$y^* = y_t + \theta_1 \ y_{-1}^* + \ldots .. + \theta_q y_{t-q}^* \ , \quad y_r^* = 0 \quad for \quad r = 0, -1, \ldots . -q$$

$$x_t^* = x_t + \theta_1 x_{t-1}^* + \ldots .. + \theta_q x_{t-q}^* \ , \quad x_r^* = 0, r = 0, -1, \ldots . - q$$

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

Z^* is a (Txq) matrix such that

$$z^*_{tr} = \theta_1 \; z^*_{-1} + \ldots . + \theta_q \; z_{t-q\,r}$$

Where, for $r = r^1 = 0, -1, \ldots ., -(q-1)$

$$z_{rr^1} = -1 \, if \;\; r = r^1$$

$$= \; 0 \, , \;\; if \;\;\; r \neq r^1$$

It can be observed that many existing techniques of estimating the model MA (1) are difficult to be used in practice for the case of MA (q) process.

V. ESTIMATORS FOR AUTOCORRELATION COEFFICIENTS AND PARAMETERS OF MOVING AVERAGE PROCESSES:

(A) LINEAR REGRESSION MODEL WITH MA (1) PROCESS FOR DISTURBANCES:

Consider the linear regression model

$$Y_{Tx1} = X_{Txk} \; \beta_{kx1} + \in_{Tx1} \tag{5.1}$$

with $\in_t = u_t - \theta_1 u_{t-1}$ (5.2)

The first order autocorrelation coefficient is given by

$$\rho_1 = \frac{\text{cov}\left(\in_t \; \in_{t-1}\right)}{\text{var}\left(\in_t\right)} = - \; \frac{\theta_1}{1+\theta_1^2} \tag{5.3}$$

$$\Rightarrow \rho_1 \theta_1^2 + \theta_1 + \rho_1 = 0 \tag{5.4}$$

The two roots of (6.7.4) are given by

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

$$\theta_1 = \frac{-1 \pm \sqrt{1-4\rho_1^2}}{2\rho_1} \tag{5.5}$$

Using the Internally Studentized residuals, an estimate of ρ_1 is given by

$$\rho_1^* = \frac{\sum_{t=2}^{T} e_t^* e_{t-1}^*}{\sum_{t=1}^{T} e_t^{*2}} \tag{5.6}$$

Where e_t^* is the t[th] element of Internally Studentized residual vector e^*.

Substituting ρ_1^* for ρ_1 in the equation (5.5), gives an estimator for parameter of MA (1) processes as.

$$\left.\begin{aligned} &\theta_1^* = \frac{-1+\sqrt{1-4\,\rho_1^{*2}}}{2\rho_1^*} \quad if \quad \left|\rho_1^*\right| < 0.5 \\ &\theta_1^* = -1 \quad , \quad if\ \rho_1^* \geq 0.5 \\ &\theta_1^* = 1 \quad , \quad if\ \rho_1^* \leq -0.5 \end{aligned}\right\} \tag{5.7}$$

If x does not contained logged dependent variable, ρ_1^* *and* θ_1^* will be consistent estimations of ρ_1 *and* θ_1 respectively. It is meaningful if $\left|\hat{\rho}_1\right| < 0.5$. Generally the small sample properties of the inequality restricted estimator θ_1^* in (5.7) and hence $\tilde{\beta}$ based on θ_1^* is difficult. It is not easy to extend estimator θ_1^* given in (5.7) to higher order moving average processes.

(B). Linear Regression Model with MA (2) Process for Disturbances:

Consider the linear regression model

$$y_{Tx1} = X_{Txk}\ \beta_{kx1} + \in_{Tx1}$$

$$\text{with } \in_t = u_t - \theta_1\ u_{t-1} - \theta_2\ u_{t-2} \tag{5.8}$$

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

the autocorrelation coefficients are given by

$$\rho_1 = \frac{-\theta_1 + \theta_1 \theta_2}{1+\theta_1^2+\theta_2^2} \tag{5.9}$$

$$\rho_2 = \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} \tag{5.10}$$

$$\rho_k = 0, \quad k > 2$$

$$\Rightarrow \eta = \frac{\rho_1}{\rho_2} = \frac{-\theta_1\left(1-\theta_2\right)}{-\theta_2} \tag{5.11}$$

Using the Internally studentized residuals, the ratio parameter can be estimated by

$$\eta^* = \frac{\sum_{t=2}^{T} e_t^* e_{t-1}^*}{\sum_{t=3}^{T} e_t^* e_{t-2}^*} \tag{5.12}$$

Also, use an estimator θ_1^* in (5.7) for θ_1 and by substituting η^* *and* θ_1^* in (5.11), one may obtain an estimator θ_2^* for θ_2 as

$$\eta^* = \frac{\theta_1^*\left(1-\theta_2^*\right)}{\theta_2^*} \tag{5.13}$$

$$\eta^* \theta_2^* = \theta_1^* - \theta_1^* \theta_2^*$$

$$\Rightarrow \theta_2^*\left(\eta^* + \theta_1^*\right) = \theta_1^*$$

$$\text{or } \theta_2^* = \frac{\theta_1^*}{\eta^* + \theta_1^*} \tag{5.14}$$

(C) Linear Regression model with MA (q) process for disturbances:

Consider the linear regression model

$$Y_{Tx1} = X_{Txk}\, \beta_{kx1} + \epsilon_{Tx1}$$

$$\in_t = u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \ldots\ldots - \theta_q u_{t-q} \tag{5.15}$$

The Autocorrelation coefficients are given by

$$\rho_k = \frac{-\theta_k + \theta_1 \theta_{k+1} + \theta_2 \theta_{k+2} + \ldots . + \theta_{q-k}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \ldots\ldots + \theta_q^2} \ , \ k = 1, 2, \ \ldots . \ q \tag{5.16}$$

$$= \quad 0 \ , \ k > q$$

Consider, $$\phi = \frac{\rho_1}{\rho_q} = \frac{-\theta_1 + \theta_1 \theta_2 + \theta_2 \theta_3 + \ldots\ldots + \theta_{q-1}\theta_q}{-\theta_q} \tag{5.17}$$

Using Internally studentized residuals, the ratio parameter can be estimated by

$$\phi^* = \frac{\sum_{t=2}^{T} e_t^* \, e_{t-1}^*}{\sum_{t=q+1}^{T} e_t^* \, e_{t-q}} \tag{5.18}$$

First obtain the estimators for $\theta_1, \theta_2, \ldots, \theta_{q-1}$ as $\theta_1^*, \theta_2^*, \ldots \theta_{q-1}^*$ respectively by estimating the parameters of MA (1), MA (2), ……, MA (q-1) processes. Then, substituting $\phi^*, \theta_1^*, \theta_2^*, \ldots, \theta_{q-1}^*$ in (5.17) yields an estimator ϕ_2^* for θ_q .

$$\phi^* = \frac{-\theta_1^* + \theta_1^*\theta_2^* + \theta_2^*\theta_3^* + \ldots\ldots + \theta_{q-1}^* \, \theta_q^*}{-\theta_q^*}$$

$$\Rightarrow -\theta_q^*\phi^* = -\theta_1^* + \theta_1^*\theta_2^* + \theta_2^*\theta_3^* + \ldots\ldots + \theta_{q-1}^* \quad \theta_q^*$$

$$\Rightarrow \theta_q^* = \frac{\theta_1^* - \theta_1^*\theta_2^* - \theta_2^*\theta_3^* - \cdots - \theta_{q-2}^* \ \ \theta_{q-1}^*}{\phi^* + \theta_{q-1}^*} \tag{5.19}$$

VI. LINEAR REGRESSION FORECASTING MODEL WITH MOVING AVERAGE PROCESS DISTURBANCES:

A) Forecasting with MA(1) Process for Disturbances:

Consider the linear regression model with MA(1) process for disturbances as.

$$y_t = x_t^1 \beta + \in_t \tag{6.1}$$

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

$$\in_t = u_t - \theta_1 u_{t-1} \ , \quad t = 1, 2, \ldots \ldots, T \qquad (6.2)$$

With $\left|\theta_1\right| < 1$ and $\mathrm{E}(U) = 0$ *and* $\mathrm{E}\left(UU^1\right) = \sigma_u^2 I_T$

Write the model (6.1) in the matrix notation as

$$Y_{Tx1} = X_{Txk}\, \beta_{kx1} + \in_{Tx1}$$

$$\mathrm{E}\left(\in\right) = 0 \ \ and \ \ \mathrm{E}\left(\in\in^1\right) = \Omega = \sigma_u^2 V \qquad (6.3)$$

$$\text{Where } V = \begin{bmatrix} 1+\theta_1^2 & -\theta_1 & 0 & \ldots\ldots & 0 & 0 \\ -\theta_1 & 1+\theta_1^2 & -\theta & \ldots\ldots & 0 & 0 \\ . & & & & . & . \\ & & & & . & . \\ & & & & . & . \\ & & & & . & . \\ 0 & 0 & 0 & \ldots\ldots & 1+\theta_1^2 & -\theta_1 \\ 0 & 0 & 0 & \ldots\ldots & -\theta_1 & 1+\theta_1^2 \end{bmatrix} \qquad (6.4)$$

By substituting θ_1^* from (5.7) for θ_1 in (2.4) and

$$\hat{\sigma}_u^2 = \frac{\sum_{t=1}^{T} e_t^2}{n-k} \qquad (6.5)$$

Where $e_{t's}$ are OLS residuals for σ_u^2 in (2.3), one may obtain an estimator for Ω as Ω^*.

The estimated GLS estimator for β is given by

$$\tilde{\beta}^* = \left[X^1 \Omega^{*^{-1}} X\right]^{-1} X^1 \Omega^{-1} Y \qquad (6.6)$$

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

and $\text{var}\left(\tilde{\beta}^{*}\right)=\left[X^{1}\Omega^{-1}X\right]^{-1}$ (6.7)

Now, the optimal point forecast of $y_{t,T+1}$ by using MA (1) process is given by

$$\tilde{y}_{t,T+1}=x_{t,T+1}^{1}\ \tilde{\beta}^{*} \quad (6.8)$$

The forecast variance is given by

$$\text{var}\left(\tilde{y}_{t,T+1}\right)=x_{t,T+1}^{1}\left[X^{1}\Omega^{*-1}X\right]^{-1}x_{t,T+1} \quad (6.9)$$

The $100\left(1-\alpha\right)\%$ confidence interval for optimal forecast is given by

$$\tilde{y}_{t,T+1}\pm\tau_{\alpha}\sqrt{x_{t,T+1}^{1}\left[X^{1}\Omega^{*-1}X\right]^{-1}x_{t,T+1}} \quad (6.10)$$

Where τ_{α} is the critical value of student's t-statistic for (T-k) degrees of freedom at $\alpha\%$ level of significance.

B) Forecasting with Ma (2) process disturbances

Consider the linear regression model with MA (2) process for disturbances as

$$y_{t}=x_{t}^{1}\beta+\in_{t}$$

$$\in_{t}=u_{t}-\theta_{1}u_{t-1}-\theta_{2}u_{t-2},t-1,2......T. \quad (6.11)$$

with $\left|\theta_{1}\right|<1,\left|\theta_{2}\right|<1,E\left(U\right)=0\,and\ E\left(UU^{1}\right)=\sigma_{u}^{2}I_{T.}$

write the model (6.11) in the matrix notation as

$$Y_{T+1}=X_{Txk}\,\beta_{kx1}+\in_{Tx1}$$

$$E\left(\in\right)=0\ \text{and}\ E\left(\in\in^{1}\right)=\Omega \quad (6.12)$$

Where $\Omega=\left(\left(w_{ij}\right)\right)$, i, j = 1, 2…. T.

Here, $w_{ij}=\gamma_{|i-j|}$ and the autocovariance functions for MA (2) process are given by

ISSN: 0975- 3583, 0976-2833 VOL 12, ISSUE 03, 2021

$$\gamma_0 = \sigma_u^2\left(1+\theta_1^2+\theta_2^2\right)$$

$$\gamma_1 = \sigma_u^2\left(-\theta_1+\theta_1\theta_2\right) \qquad (6.13)$$

$$\gamma_2 = -\sigma_u^2\theta_2$$

$$\gamma_k = 0, k > 2$$

By substituting θ_1^* from (5.7), θ_2^* from (5.14) and $\hat{\sigma}_u^2$ from (6.5) in the relations (6.13), one may obtain an estimator for Ω as $\Omega_{(2)}^*$

The estimated GLS estimator for β is given by

$$\tilde{\beta}^*(2) = \left[X^1\Omega_{(2)}^{*-1}X\right]X^1\Omega_{(2)}^{*-1}Y \qquad (6.14)$$

and $Var\left(\tilde{\beta}^*(2)\right) = \left[X^1\Omega_{(2)}^{*-1}X\right]^{-1}$ (6.15)

Now an optimal point foreca1st of $y_{t,T+1}$ by using MA(2) process is given by

$$\tilde{y}_{t,T+1}(2) = x_{t,T+1}^1\,\tilde{\beta}^*(2) \qquad (6.16)$$

The Forecast variance is given by

$$\text{var}\left(\tilde{y}_{t,T+1}(2)\right) = x_{t,T+1}^1\left[X^1\Omega_{(2)}^{*-1}X\right]^{-1}x_{t,T+1} \qquad (6.17)$$

C). FORECASTING WITH MA(q) PROCESS DISTURBANCES:

Consider the linear regression model with MA (q) process for disturbances as

$$y_t = x_t^1\beta + \in_t$$

$$\in_t = u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \theta_3 u_{t-3} - \ldots\ldots - \theta_q u_{t-q} \qquad t = 1, 2, \ldots. T \qquad (6.18)$$

with $\left|\theta_i\right| < 1, \forall\, i = 1, 2, \ldots. q$, $E(U) = 0$

and $E(UU^1) = \sigma_u^2\, I_T$

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Write the model (6.18) in the matrix notation as

$$Y_{Tx1} = X_{Txk}\ \beta_{kx1} + \in_{Tx1}$$

$$E(\in) = 0 \ \ and \ E\left(\in\in^{1}\right) = \Omega \qquad (6.19)$$

$$\text{Where } \Omega = \begin{bmatrix} . & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & \gamma_q & . \\ . & . & . & . & . & \gamma_1 & . & . \\ . & . & . & . & \gamma_0 & . & . & . \\ . & . & . & \gamma_1 & . & . & . & . \\ . & . & \gamma_q & . & . & . & . & . \\ 0 & . & . & . & . & . & . & . \end{bmatrix} \qquad (6.20)$$

Here, $\gamma_0, \gamma_1, \gamma_2, \ldots, \gamma_q$ are the autocovariance functions of MA (q) process.

We have, $\gamma_0 = \sigma_u^2\left(1+\theta_1^2+\theta_2^2+\ldots+\theta_q^2\right)$

$$\gamma_k = \sigma_u^2\left(-\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + \ldots + \theta_{q-k}\theta_q\right) \quad k = 1, 2, \ldots q$$

$$\gamma_s = 0\ , \ \ for \ \ s > q$$

By substituting the estimates $\theta_1^*, \theta_2^*, \ldots, \theta_q^*$ as given in (C) for $\theta_1, \theta_2, \ldots, \theta_q$, one may obtain an estimate for Ω as $\Omega^*(q)$.

The estimated GLS estimator for β is given by

$$\tilde{\beta}^*(q) = \left[X^1\Omega^{*^{-1}}(q)X\right]^{-1} \quad X^1\Omega^{*^{-1}}(q)y \qquad (6.21)$$

$$\text{and } \operatorname{var}\left(\tilde{\beta}^*(q)\right) = \left[X^1\Omega^{*^{-1}}(q)X\right]^{-1} \qquad (6.22)$$

Now, an optimal point forecast of $y_{t,T+1}$ by using MA (q) process is given by

$$\tilde{y}_{t,T+1}(q) = x^{1}_{t,T+1} \ \tilde{\beta}^{*}(q) \tag{6.23}$$

The forecast variance is given by

$$\operatorname{var}\left(\tilde{y}_{t,T+1}(q)\right) = x^{1}_{t,T+1} \left[X^{1}\Omega^{*-1}(q)X\right]^{-1} x_{t,T+1} \tag{6.24}$$

VII. CONCLUSIONS:

In forecasting models, linear regression models are widely used, attention is concerned on model specification when the model is taken to be linear.

In view of the importance of forecasting in empirical research, some new procedures for Applied Forecasting have been proposed in the present study.

In the present research work, the forecasting techniques have been developed by using Internally studentized residuals.

Linear regression models with first, second and q^{th} order moving average processes of disturbances have been specified and GLS estimators are developed for the parameters of the linear regression model. The autocovariance functions and autocorrelation coefficients have been given for these moving average processes.

Estimators for autocorrelation coefficients and parameters of first, second and q^{th} order moving average processes for disturbances have been derived by using internally studentized residuals. A recursive procedure has been applied to obtain estimators of the parameters of general q^{th} order moving average process.

The estimated linear regression models with moving average processes for disturbances have been implemented for the purpose of forecasting. The confidence intervals for the optimal forecasts along with the forecast variance have also been presented in the study.

This kind of research can be further extended by specifying linear and nonlinear regression models with autoregressive / moving average processes for disturbances. The proposed estimation method can be applied to sets of linear regression models or seemingly unrelated regression equation (SURE) models with autoregressive / moving average processes for disturbances and then it can be used for the purpose of forecasting.

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